JW-003-1015043 Seat No. ________

B. Sc. (Sem. V) (CBCS) (W.I.F.-2016) Examination

October - 2019

Statistics : S-502

[Mathematical Statistics]

(New Course)

Faculty Code : 003
Subject Code : 1015043

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) All questions carry equal marks.
(3) Student can use their own scientific calculator.

1 (a) Give the answer of following questions : **4**

(1) _____ is a characteristic function of Standard Normal distribution.

(2) _____ is a characteristic function of Geometric distribution.

(3) _____ is a characteristic function of Poisson distribution.

(4) _____ is a characteristic function of chi-square distribution.

(b) Wirte any one : **2**

(1) Obtain characteristic function of Binomial distribution.

(2) Show that $\varnothing_x(0)=1$.

(c) Write any one : **3**

(1) Obtain characteristic function of Normal distribution.

(2) Obtain probability density function for the characteristic function $\varnothing_x(t)=e^{-\left(\frac{t^2\sigma^2}{2}\right)}$.

(d) Write any one : **5**

(1) State and prove weak law of large number.

(2) Prove that

(i) $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_x(t) \right]_{t=0}$

(ii) $\mu_r = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_u(t) \right]_{t=0}$; where $u = x - \mu$.

2 (a) Give the answer of following questions : **4**

(1) Measured of Kurtosis coefficient for Normal distribution are _____ and _____ .

(2) For normal distribution Mean deviation = _______.

(3) For Normal distribution $\mu_{2n} =$ _______.

(4) If two independent variates $X_1 \sim N\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu_2, \sigma_2^2\right)$ then $X_1 - X_2$ is distributed as _____.

(b) Write any one : **2**

(1) Obtain CGF of Normal distribution and from it show that $\mu_4 = 3\sigma^4$.

(2) Obtain median of Normal distribution.

(c) Write any one : **3**

(1) Show that a linear combination of independent normal variates is also normal variate.

(2) Obtain mode of Normal distribution.

(d) Write any one : **5**

(1) Derive Normal distribution.

(2) Obtain MFG of normal distribution and also show that $\beta_1 = 0$ and $\beta_2 = 3$.

3 (a) Give the answer of following questions : **4**

(1) If two independent variates $X_1 \sim \Lambda\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim \Lambda\left(\mu_2, \sigma_2^2\right)$ then $X_1 \div X_2$ is distributed as _____.

(2) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1 + X_2}$ is distributed as _____.

(3) ______ is a moment generating function of $\gamma(\alpha, p)$.

(4) Weibull distribution has application in ______.

(b) Write any one : **2**

(1) Define gamma distribution and find its mean.

(2) Define uniform distribution and find its mean.

(c) Write any one : **3**

(1) Obtain the relation between gamma and normal distribution.

(2) Define beta distribution of first kind and find its mean and variance.

(d) Write any one : **5**

(1) Obtain MGF of Gamma distribution with parameters α and p. Also show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(2) Obtain coefficient of skewness for log standard normal distribution.

4 (a) Give the answer of following questions : **4**

(1) The mean of the chi-square distribution is _____ of its variance.

(2) $t-$distribution curve in respect of tails is always _____.

(3) If two independent variates $X_1 \sim \Lambda(\mu_1, \sigma_1^2)$ and $X_2 \sim \Lambda(\mu_2, \sigma_2^2)$ then $X_1 \cdot X_2$ is distributed as _____.

(4) $t-$distribution with 1 d.f. reduces to _____.

(b) Write any one : **2**

(1) Obtain MGF of χ^2 distribution.

(2) Obtain relation between t-distribution and F-distribution.

(c) Write any one : **3**

(1) Obtain CGF of χ^2 distribution and show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(2) Obtain limiting form of t-distribution for large degrees of freedom.

(d) Write any one : **5**

(1) Derive t-distribution.

(2) Derive F-distribution.

5 (a) Give the answer of following questions : **4**

(1) Multiple correlation is a measure of _______ association of a variable with other variables.

(2) If $r_{12} = 0.28, r_{23} = 0.49, r_{31} = 0.51,$ $\sigma_1 = 2.7, \sigma_2 = 2.4, \sigma_3 = 2.7$ then $b_{31.2} =$ ______.

(3) The range of partial correlation coefficient is _____.

(4) Partial correlation coefficients is a measure of association between two variables _______ the common effect of the rest of the variable.

(b) Write any one : **2**

(1) Usual notation prove that $\sigma_{1.23}^2 = \sigma_1^2 \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$.

(2) Obtain μ_{20} for bivariate normal distribution.

(c) Write any one : **3**

(1) Usual notation of multiple correlation and multiple regression, prove that $b_{12} = \dfrac{b_{12.3} + b_{13.2}\, b_{32.1}}{1 - b_{13.2}\, b_{31.2}}$.

(2) Obtain conditional distribution of x when y is given for Bi-variate distribution.

(d) Write any one : **5**

(1) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \dfrac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}$.

(2) Obtain marginal distribution of y for Bi-variate distribution.